STUDIES ON SOME ERROR PRONE ATTRIBUTE SAMPLING INSPECTION PLANS

Pranjal Kr. Chakravartty
Data Analyst, ARIAS Society, Guwahati, an Autonomous body of Govt. of Assam
Email: khukustar@googlemail.com

Abstract

Attribute sampling is a widely used method in quality control and inspection to determine the acceptability of a batch or lot of items based on the presence or absence of certain attributes, such as defects or non-conformities. The goal is typically to make decision about whether to accept or reject the entire batch based on a sample of items. The advantage of the single sampling plan is its simplicity and ease of use. There have been several studies conducted on error-prone attribute sampling inspection plans, including the Single Sampling Plan, Double Sampling Plan, Semi-curtailed Sampling Plan, and Repetitive Group Sampling Plan. Many studies have explored various aspects of those plans, including their validity, efficiency, and performance in detecting and controlling errors in attribute sampling inspections. In this paper, misclassification error, attribute sampling plans, and characteristics along with OC curve of these plans have been discussed. Besides, ASN, AOQ, ATI curves also plotted for these attribute sampling plans.

Key words: Inspection Error, Single, Double, Semi-curtailed and Repetitive Group Sampling Plan

1. INTRODUCTION

The problem of dealing misclassification as well as measurement error (inspection errors) while analyzing Statistical data in various subjects of study is significantly wide in scope. Cochran [12] reviewed some of the mathematical models used to study the extent to which standard techniques of analysis become erroneous and misleading if certain types of errors of measurement are present. Similarly, the effect of inspection error is highly significant and had drawn attention to many quality scientists for last few decades in the area of Statistical Quality Control (SQC). It is possible that statisticians in many practical situations tend to forget effects of such inspection error in their day to day works. These sometime severely effects the quality objective to be achieved by using SQC tools like control charts and acceptance sampling plans (both by attributes and variables). Even under what may be considered as an inspection condition, it is not uncommon to find error figures of 25% or more even for the most experienced inspector personnel Collins et.al. [14]. To study the effect of Type I and Type II inspection errors on the characteristics of various sampling inspection plans by attributes we first give the formula of observed defective developed by Freeman et.al. [23].

1.1 Development of formula for observed fraction defective

In a sampling inspection scheme, two types of inspection errors may occur. An item which is actually non defective, one may classify it as a defective one (Type I Error) and an item which is actually defective one may classify it is as a non-defective one (Type II Error). The formula for observed fraction defective can be developed in the following manner:

Let,

E_1 = the event that a non-defective item is classified as a defective

E_2 = the event that a defective item is classified as a non-defective

A = the event that an item is defective

and

B = the event that an item is classified as defective

If we define

$p = P(A)$, the true fraction defective

$p_e = P(B)$, the observed fraction defective

$e_1 = P(E_1)$, the probability that E_1 occurs

and $e_2 = P(E_2)$, the probability that E_2 occurs

Then $P(B) = P(A)P(\bar{E}_2) + P(\bar{A})P(E_1)$, that is the expression for the observed fraction defective p_e may be expressed as

$$p_e = p(1-e_2)+(1-p)e_1$$

$$= e_1 + (1-e_1-e_2)p \tag{1.1.1}$$

When $e_1 = 0 = e_2$, then $p = p_e$,

where p is the true fraction defective, i.e. proportion of incoming items which is actually defective.

2. PAST WORKS OF INSPECTION ERROR ON ATTRIBUTE SAMPLING PLANS

Ayoub et. al. [1], recommended that complete theoretical development of the area of quality control under inspection and instrument error is essential. Even before Ayoub et. al.'s recommendations, a good number of researchers have developed models for attribute sampling plan under the assumption that inspection tasks are not free from error. Such concept of imperfect inspection was originated by Juran [30, 31], Freeman et.al. [23], Jacobson [28] and among others.

Singh [48], derived the probabilities of misclassification due to measurement error in an attribute sampling plan. He described a method of obtaining the true operating characteristic (OC) and Average Outgoing Quality (AOQ) curve for a single sampling plan for attributes. An investigation is also made by him to find the sample size required for accepting a product with desired confidence, considering particular values of true and observed fraction defective. The effect of inspection error on the design of attributes acceptance sampling plan has been also considered by Davis [16] and McKinght [35]. They discussed the effect of inspection error on the performance measure of AOQ function when items which appear to be defective are replaced assuming perfect inspection of replacement items. They also considered the performance measure Average Total Inspection (ATI) when items which appear to be defective are removed from sampled lot but are not replaced. Ayoub et.al. [2], investigated the effect of inspection error on double and sequential sampling plan by attributes. Minton [36], investigated the effect of measurement error on single sampling by attribute. He assessed the formula for acceptable and unacceptable quality level, AOQ and power of the sampling plan in presence of inspection error. Collins et.al. [13], considered the effect of inspection error on the probability of acceptance, AOQ and ATI of a single sampling by attributes under both replacement and non-replacement situations. They presented a method for designing an acceptance plan in presence of inspection error. Collins et.al. [14], Case et.al. [8], studied the Dodge's continuous sampling plan (CSP-1) under inspection error. Bennett et.al. [4], considered the problem of economic design of single acceptance sampling by attributes in presence of inspection errors assuming prior distribution of incoming lot quality. Case et.al. [9], also developed formula for calculating the AOQ under type-I and type-II inspection errors. Nine different rectification schemes were considered by them. Effect of inspection error on sequential sampling plan and their compensating formulae has also been considered by Kulatrapa and Case [33]. Singh and Dutta [49], reviewed with brief critical analysis of the major past works in the area of measurement and misclassification errors in Statistical Quality Control. Dutta [19], derived the formula for OQC, OAOQ and OASN functions for double sampling plan and compensating the specified value of producer's risk and consumer's risk. Dutta [20], studied the effect of inspection error on the characteristics of the curtailed single sampling plan and gave the compensating formula for the sample size. He (Dutta, [20]) also extended his study to investigate the effect of misclassification errors on the characteristics of Independent and Generalized Dependent Mixed plans. Bennett et. al. [5], studied the inspection error on a cost-based single sampling plan. Dorris and Foote [17], made a survey on papers dealing with inspection errors and Statistical Quality Control and point out shortcomings of existing approaches and suggested future line of investigation. Carr [7], estimated the inspection error due to misclassification and gave the formula for compensating the OC and AOQ curves of skip-lot plans and sampling plans for rejected lot that have been screened hundred percent. Hassan and Manaspiti [27], derived the interrelated system of manufacturing and assembly quality levels and average incoming quality levels using dynamic program in error prone semi-economic Lot-by-Lot plan and Continuous Sampling (CSP-1).

Maghsoodloo and Bush [34], investigated the characteristics of error prone double sampling by attributes almost in a similar manner to Dutta [19]. However, they have extended their study to investigate the characteristics of the sampling plan for both replacement and non-replacement environment. Trader and Huss [50], examined the effect of inspection errors on Bayesian model of Binomial and Hyper geometric distributions. Ghosh [25], studied the effect of inspection error on AOQ functions for single sampling rectification plan. Jaraiedi et. al. [29], investigated the posterior analysis in inspection accuracy. Chakraborty and Rathe [10], used GERT analysis technique to analyze inspection error in continuous sampling plan. Bagchi [3] discussed the dependence of producer's risk α and consumer's risk β under the influence of inspection error where the lots are accepted or rejected based on attribute sequential sampling plan. Nandi et. al. [38] investigated the effect of inspection error on the characteristics of Repetitive Group Sampling plan of Sherman [47]. Dutta [22] investigated the characteristics of Generalized Dependent Mixed (GDM) plans in presence of measurement error. Sarma [45] made a critical analysis of various error prone models of control charts & acceptance sampling plans for attributes. He also extended the study for variable control charts as well as for sampling plans by variables when the observations are subjected to measurement error. Duffuaa [18], proposed an error prone repeat inspection plan for the controlling the quality of critical multi characteristic components. All the models have been developed assuming type-I and type-II errors to be known and fixed. He derived the Receiver's Operating Characteristic (ROC) curve, Signal Detection Theory (SDT) and Contrained Regression (CR) for obtaining a function that relates to incoming quality.

Khan and Duffuaa [32], studied the impact of inspection errors on single, double and repeat inspection plans. They recommended some measures to guard against the effect of inspection errors and develop a better strategy to control it.

Gao and Tang [24], developed a mathematical model to investigate error prone characteristics of chain sampling plan. They found that, the effect of Type-II errors on the probability of acceptance is very marginal compared to that of Type-I error when the true fraction non-conforming is small. Chen C-Ho [11], gave a modified version of Pulak and Al-Sultan's [40] model for determining economic specification limit of error prone single sampling rectifying inspection scheme. He applied Taguchi's systematic quadratic loss function in evaluating the product quality. Some of the recent works in this area were also reviewed by him. Radhakrishnan and Ravi Sankar [41] studied the "Three classes Attributes Double Sampling Plan indexed through Maximum Allowable Average Outgoing Quality (MAAOQ)". This 3-class attribute sampling procedure classifies items in a lot under consideration into three categories of quality good, marginal and bad. Radhakrishnan and Esther [42], studied the procedure for constructing tightened continuous sampling plan-3 indexed through the convex combination of Average Outgoing Quality Limit (AOQL) and Maximum Allowable Average Outgoing Quality (MAAOQ). Schorn [46], developed a model of the cost and quality effectiveness of single and multiple 100% inspections and applies that model to management decision making in the foundry. They evaluated four specific cost cases against a series of potential scenarios of visual inspection error and the number of inspections performed. Kumar, Kiruthika and Radhakrishnan [44], studied the "Construction of Mixed Sampling Plans indexed through MAPD and AQL with Conditional Double Sampling Plan as Attribute Plan using weighted Poisson distribution" and presented the baseline distribution. It is concluded from the study that the second sample size required for mixed sampling plan with conditional double sampling plan as attribute plan indexed through MAPD is less than that of the second stage sample size of the mixed sampling plan with conditional double sampling plan as attribute plan indexed through AQL, justified by

Kumar [43]. These plans definitely help the producers, because of the lesser sample size which directly result in lesser sampling cost and indirectly reduces the total cost of the product.

Mujahida & Singh [37] discussed a paper entitled "Mixed Sampling Plans for Markoff model under Inspection error". They studied the effects of inspection error on the OC and ASN functions for independent and dependent mixed acceptance sampling plans. In their study, for error prone variable plans random error terms are considered to be according to Markoff model for coefficient of variation (CV) and attribute sampling plans analysis with regard to the choice of a sampling plan taking inspection error into consideration. A comparison between the independent and dependent mixed plan have been made in their paper in respect of OC and ASN functions under inspection error. There studies are almost similar to that of Dutta [20].

3. SOME ERROR PRONE SAMPLING PLAN MODELS

3.1 Single Sampling Plan

It has been shown by Hald [26] that the Binomial Distribution is reproducible to hyper-geometric sampling. That is if a lot of size N it formed from a Binomial data generating process and a sample of p items is taken from the lot, then, the number of defective in the sample is described by a Binomial mass function. Thus, assuming perfect inspection, the probability lot acceptance is given by

$$P_a = \sum_{d=0}^{c} \binom{n}{d} (p)^d (1-p)^{n-d} \tag{1.3.1}$$

As a function of lot quality p, P_a determines the sampling plans Operating Characteristic (OC) curve.

The error prone probability of acceptance P_a, when inspection error is present, can be written in a similar manner by replacing the true fraction defective p equation (1.3.1) by the observed fraction defective p_e of (1.1.1), which gives

$$P_{a_e} = \sum_{d=0}^{c} \binom{n}{d} (p_e)^d (1-p_e)^{n-d} \tag{1.3.2}$$

(a) Probability of Acceptance of Double Sampling by Attribute under Inspection Error

The usual double sampling plan by attribute is described as follows:

A sample of size n_1 is drawn from the lot submitted for inspection and number of defectives d_1 are determined. If $d_1 \le c_1$; the lot is accepted. If $d_1 > c_2$, the lot is rejected. If $c_1 < d_1 \le c_2$, a second sample of size n_2 is drawn and the number of defective d_2 is noted. If $d_1 + d_2 \le c_2$, the lot is finally accepted. If $d_1 + d_2 > c_2$, the lot is finally rejected. The formula for OC function of double sampling plan for the true fraction defective p is given by Bowker and Lieberman (1972) as

$$Pa = \sum_{d_1=0}^{c_1} \binom{n_1}{d_1}(p)^{d_1}(1-p)^{n_1-d_1} + \sum_{d_1=c_1+1}^{c_2} \sum_{d_2=0}^{c_2-d_1} \binom{n_1}{d_1}(p)^{d_1}\binom{n_2}{d_2}(p)^{d_2}(1-p)^{n_2-d_2} \qquad (1.3.3)$$

where c_1 and c_2 are the acceptance numbers for first and combined samples respectively.

The probability of acceptance p_{a_e}, the observed OC of DSP is written in a similar manner by Dutta [19] who developed the error prone DSP as

$$P_{a_e} = \sum_{d_1=0}^{c_1} \frac{\exp^{-n_1 p_e}(n_1 p_e)^{d_1}}{d_1!} + \frac{\exp^{-n_1 p_e}(n_1 p_e)^{c_1+1}}{(c_1+1)!} .[\sum_{d_2=0}^{c_2-c_1-1} \frac{\exp^{-n_2 p_e}(n_2 p_e)^{d_2}}{d_2!}]$$

$$+ \frac{\exp^{-n_1 p_e}(n_1 p_e)^{c_1+2}}{(c_1+2)!} .[\sum_{d_2=0}^{c_2-c_1-1} + \frac{\exp^{-n_2 p_e}(n_2 p_e)^{d_2}}{d_2!} + \ldots + \frac{\exp^{-n_1 p_e}(n_1 p_e)^{c_2}}{c_2!} .[\exp^{-n_2 p_e}] \qquad (1.3.4)$$

3.2 Probability of Acceptance for Curtailed Sampling Plan under Inspection Error

The nomenclature and use of curtail sampling was first put forth by the Freeman et. al, [23] of Columbia University. They however, dealt with mainly a special kind of curtailed sampling plan called Semi-Curtailed plan.

In semi-curtailed plan inspection is terminated as soon as the rejection of the lot becomes a sure even due to the occurrence of enough defectives. A semi-curtailed plan can be described as follows:

(a) Semi-Curtailed Inspection Plan

Inspect randomly selected units from the lot, one unit at a time, until either *k* defectives have been observed or until *n* units have been inspected. Reject the lot as soon as *k* defectives are observed. Accept the lot if the number of defectives observed is less than *k* provided that *n* units have been inspected.

The semi-curtailed inspection plans have been further generalized to what is known as fully-curtailed sampling plans. In fully-curtailed sampling plans, inspection is terminated as soon as rejection of the lot becomes a sure event or the acceptance of the lot becomes a sure event, without waiting for inspection items. Fully-curtailed plan is described as follows:

(b) Fully-Curtailed Sampling Plan

Inspect randomly selected units from the lot, one unit at a time, until either k defectives have been observed or $n-k+1$ non-defectives have been observed. Accept the lot if there are $n-k+1$ non-defectives. Reject the lot if there are k defectives.

The OC function the most convenient expression for semi-curtailed sampling plan and fully-curtailed sampling plan are given by Craig [15] and Phatak and Shah [39] respectively as

$$\underset{(Semi)}{P_a} = (1-p)^{n-k+1} \sum_{x=0}^{k-1} \binom{n-x-1}{k-1-x} p^{k-1-x} \tag{1.3.5}$$

$$\underset{(Fully)}{P_a} = (1-p)^{n-k+1} \sum_{x=0}^{k-1} \binom{n-k+x}{n-k} p^{x} \tag{1.3.6}$$

The formula for OC function of complete Inspection, Semi-curtailed Inspection and fully-curtailed inspection given respectively by (1.3.1), (1.3.5) and (1.3.6) are identically same but for lookout. We shall however choose formula (1.3.5) for further discussion since it is most suitable for evaluation purpose especially when n is large and k is small. On considering the Type-I and Type-II error the characteristic of the curtailed sampling plans can be evaluated, using the observed OC function of Dutta [21] as

$$P_{a_e} = (1 - p_e)^{n-k+1} \sum_{x=0}^{k-1} \binom{n-x-1}{k-1-x} p_e^{\,k-1-x} \tag{1.3.7}$$

3.4 Repetitive Group Sampling Scheme

The Repetitive Group Sampling (RGS) Plan was first given by Sherman [47]. It is characterized three design variables (n, c_1, c_2) and its procedure is as follows:

(i) Take a random sample of size n.

(ii) Count the number of defectives, d in the sample

(iii) If $d \le c_1$, accept the lot

$d > c_2$, reject the lot

$c_1 < d \le c_2$, repeat step 1,2,3.

Operating characteristic curves of RGS

The probability of acceptance of lot under R.G.S. plan is the absence of inspective error is given as by Sherman [47] as:

$$p_A(p) = \frac{p_a(p)}{p_a(p) + p_r(p)} \tag{1.4.1}$$

where, $p_a(p) = \sum_{d=0}^{c_1} \binom{n}{d} p^d (1-p)^{n-d}$

$= B(c_1, n, p)$

$$p_r(p) = \sum_{d=c_2+1}^{n} \binom{n}{d} p^d (1-p)^{n-d}$$

$$= 1 - B(c_2, n, p) \tag{1.4.2}$$

Thus, the OC function under inspection errors for R.G.S. Plan given by Nandi et.al. [38] is

$$p_{A_e}(p) = \frac{p_{a_e}(p)}{p_{a_e}(p) + p_{r_e}(p)} \tag{1.4.3}$$

where,

$$p_{a_e}(p) = p_a(p_e) = B(c_1, n, p_e) \tag{1.4.4}$$

$$p_{r_e}(p) = p_r(p_e) = 1 - B(c_2, n, p_e) \tag{1.4.5}$$

For evaluation purpose error prone OC functions of Single Sampling Plan (Equation (1.3.2)) and Double Sampling Plan (equation (1.3.4)) are rewritten using well known Poisson approximation to Binomial Probability function (Dutta, [19]. The observed OC for Single, Double, Semi-Curtailed and Repetitive Group Sampling (RGS) are reproduced in Table - I as follows for ready reference:

Table 1: Error Prone OC Functions

Sampling Plan	**Observed OC for different Sampling Plan**
Observed Semi-Curtailed	$P_{a_e} = (1-p_e)^{n-k+1} \sum_{x=0}^{k-1} \binom{n-x-1}{k-1-x} p_e^{\,k-1-x}$ (1.3.7)
Repetitive Group Sampling (RGS)	$p_{A_e}(p) = \dfrac{p_{a_e}(p)}{p_{a_e}(p) + p_{r_e}(p)}$ (1.4.3)
Single	$P_{a_e} = \sum_{d=0}^{c} \dfrac{e^{-\lambda}\lambda^d}{d!}$ where $\lambda = np_e$ (1.4.6)
Double	$P_{a_e} = \sum_{d_1=0}^{c_1} \dfrac{\exp^{-n_1 p^e}(n_1 p_e)^{d_1}}{d_1!} + \dfrac{\exp^{-n_1 p_e}(n_1 p_e)^{c_1+1}}{(c_1+1)!} .[\sum_{d_2=0}^{c_2-c_1-1} \dfrac{\exp^{-n_2 p_e}(n_2 p_e) d_2}{d_2!}]$ $+ \dfrac{\exp^{-n_1 p_e}(n_1 p_e)^{c_1+2}}{(c_1+2)!} .[\sum_{d_2=0}^{c_2-c_1-1} + \dfrac{\exp^{-n_2 p_e}(n_2 p_e)^{d_2}}{d_2!} + \ldots + \dfrac{\exp^{-n_1 p_e}(n_1 p_e)^{c_2}}{c_2!} .[\exp^{-n_2 p_e}]$ (1.4.7)

With varying sizes of error of misclassification, the observed fraction defective p_e, for different incoming lot quality p is tabulated in Table - II as follows:

Table 2: Observed fraction defective p_e for some selected pairs

$e_1 = 0.0$ $e_2 = 0.0$	$e_1 = 0.005$ $e_2 = 0.0$	$e_1 = 0.0$ $e_2 = 0.25$	$e_1 = 0.005$ $e_2 = 0.25$
$p_e = p$	p_e	p_e	p_e
0.005	0.0100	0.0038	0.0087
0.01	0.0150	0.0075	0.0125
0.02	0.0249	0.0150	0.0198
0.03	0.0349	0.0225	0.0274
0.05	0.0548	0.0375	0.0423
0.07	0.0747	0.0525	0.0572
0.08	0.0846	0.0600	0.0646
0.10	0.1045	0.0750	0.0795
0.15	0.1543	0.1125	0.1168
0.20	0.2040	0.1500	0.1540

The observed OC functions for Double Sampling Plan given in Table - III. Similarly, the observed OC functions for Single Sampling Plan, Repetitive Group Sampling Plan and Semi-Curtailed sampling plan are given in Table – IV and Table - VII. The AOQ and ATI functions of R.G.S. plan and error prone AOQ for Doubled Sampling Plan are provided in Table – V and VI. The observed ASN functions for Double Sampling Plan and Semi-Curtailed sampling plan are provided in Table - VIII and Table - IX respectively.

Tabl 3: Error prone OC for Doubled Sampling Plan by Attributes for varying error sizes

$$\mathbf{N = 4000,\ n_1 = 50,\ n_2 = 100,\ c_1 = 0,\ c_2 = 3}$$

p \ P_a	$e_1 = 0.0$ $e_2 = 0.0$	$e_1 = 0.005$ $e_2 = 0.0$	$e_1 = 0.0$ $e_2 = 0.05$	$e_1 = 0.005$ $e_2 = 0.001$
0.0030	1.0000	0.9992	1.0000	0.9992
0.0050	0.9998	0.9982	0.9999	0.9982
0.0100	0.9982	0.9928	0.9985	0.9928
0.0300	0.9343	0.8979	0.9373	0.8975
0.0500	0.7576	0.7002	0.7942	0.6978
0.0800	0.4335	0.3826	0.4735	0.3826
0.1000	0.2920	0.2617	0.3020	0.2610

Observe OC plot for error pairs with

$$(e_1 = e_2 = 0), (e_1 = 0.005, e_2 = 0), (e_1 = 0, e_2 = 0.05)(e_1 = 0.005, e_2 = 0.001)$$

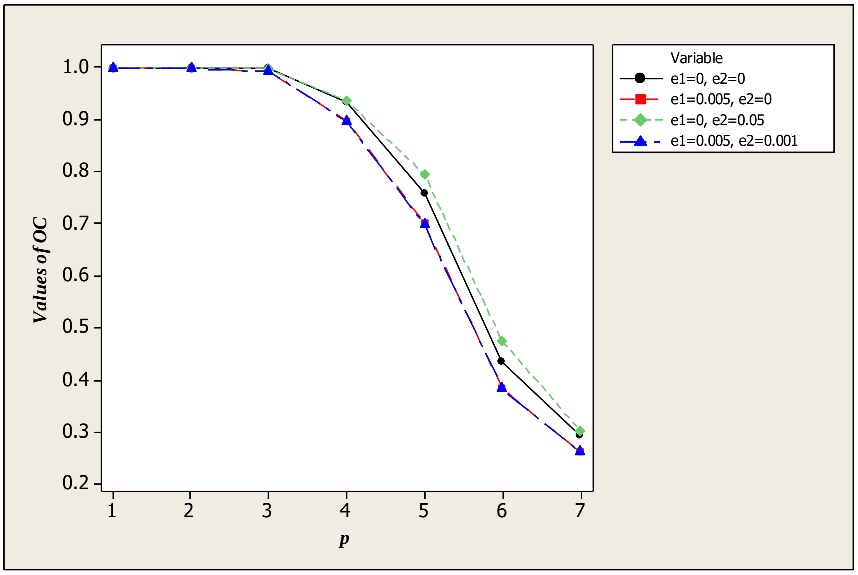

Figure 1: Double Sampling Plan curve for error prone OC

Table 4: Error prone OC of Single & R.G.S. Sampling Plan by Attributes

$$\mathbf{N = 4000, n = 100, c_1 = 2, c_2 = 5, e_1 = 0.005, e_2 = 0.02}$$

p (True)	**p_e (Observed)**	**OC (Single) Error free**	**OC (Single) Error Prone**	**OC (RGS) Error free**	**OC (RGS) Error prone**
0.005	0.0099	0.99	0.92	0.9982	0.9804
0.01	0.0148	0.92	0.82	0.9798	0.9284
0.03	0.0343	0.42	0.33	0.5454	0.4241
0.05	0.0538	0.12	0.10	0.1450	0.1131
0.07	0.0733	0.03	0.02	0.0313	0.0210
0.10	0.1025	0.003	0.0023	0.0028	0.0023

Plot of OC (Single Sampling Plan) for Error free & Error prone; $e_1 = 0.005,\ e_2 = 0.02$

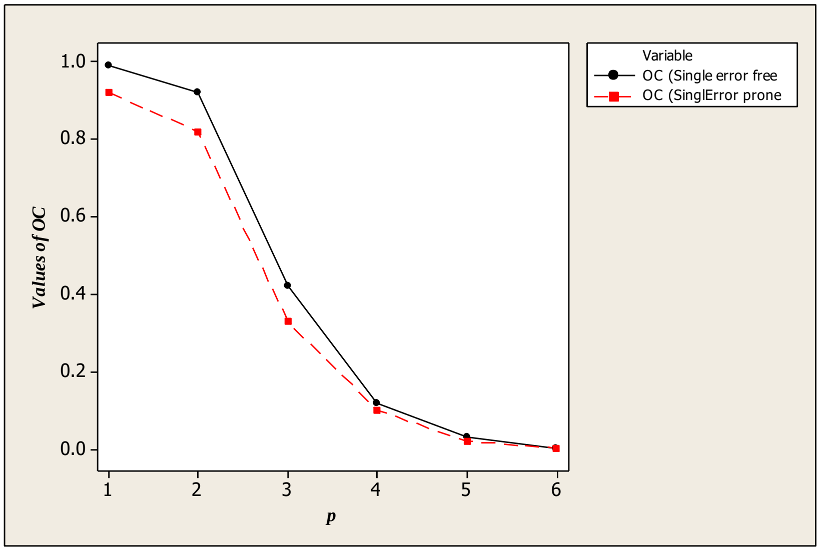

Figure 2: OC Curve for Error free and Error prone Single Sampling Plan by Attributes

Plot of OC of RGS for Error free and Error prone; e_1=0.005, e_2=0.02

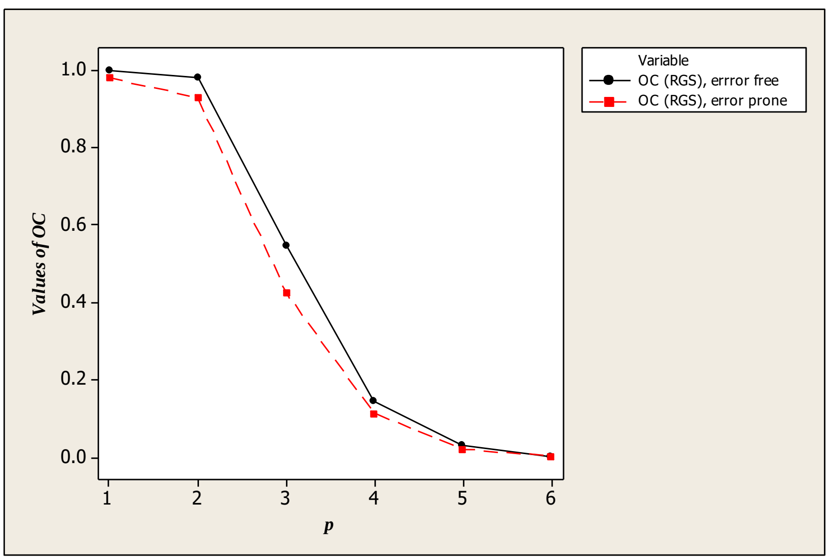

Figure 3: OC of RGS Curve for Error free and Error prone cases

Table 5: Error prone AOQ & ATI of RGS Sampling Plan by Attributes

N=4000, n=100, c_1=2, c_2=5, e_1=0.005, e_2=0.02

p (True)	p_e (Observed)	AOQ (Error free)	AOQ Error Prone	ATI Error free	ATI Error prone
0.005	0.0099	0.0050	0.0097	107.02	176.44
0.01	0.0148	0.0098	0.0137	178.78	379.24
0.03	0.0343	0.0164	0.0145	1872.94	2346.01
0.05	0.0538	0.0073	0.0061	3434.50	3558.91
0.07	0.0733	0.0022	0.0015	3877.93	3918.10
0.10	0.1025	0.0003	0.0002	3989.08	3991.03

Plot of AOQ for Error free and Error prone; e_1=0.005, e_2=0.02

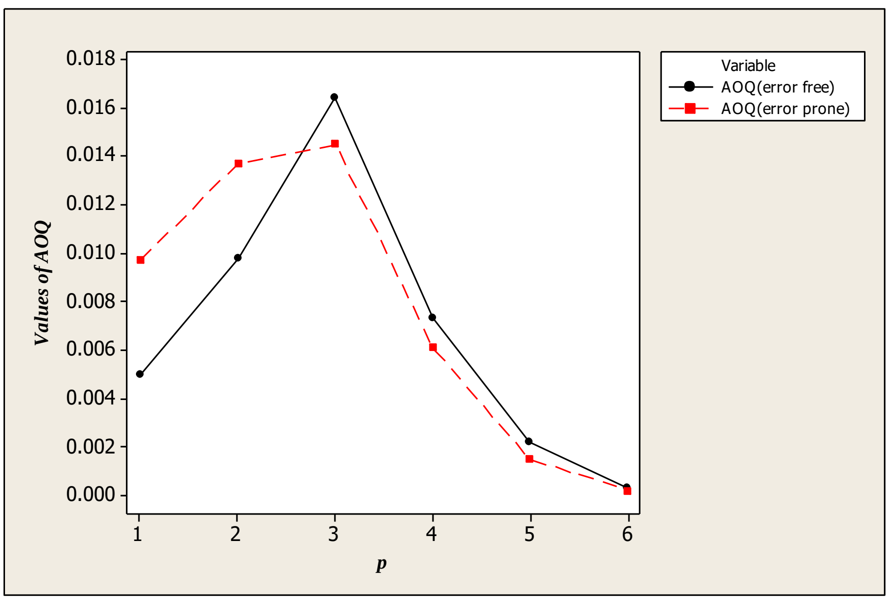

Figure 4: Observe AOQ Curve of RGS for Error free and Error prone

Plot of ATI for Error free and Error prone; e_1=0.005, e_2=0.02

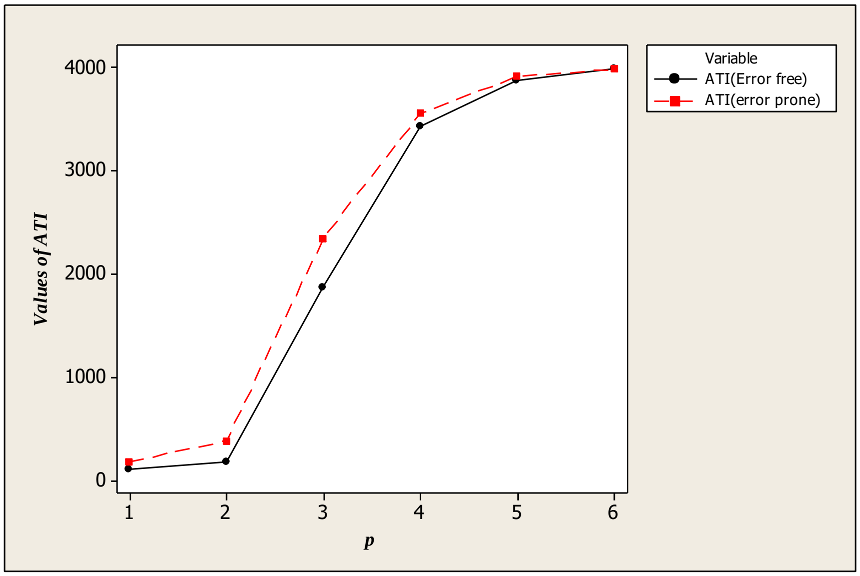

Figure 5: Observe ATI Curve of RGS for Error free and Error prone

Table 6: Error prone AOQ for Doubled Sampling Plan by Attributes for varying error sizes

N=4000, n_1=50, n_2=100, c_1=0, c_2=3

AOQ / p	$e_1 = 0.0$, $e_2 = 0.0$	$e_1 = 0.005$, $e_2 = 0.0$	$e_1 = 0.0$, $e_2 = 0.05$	$e_1 = 0.005$, $e_2 = 0.001$
0.003	0.0030	0.0030	0.0030	0.0030
0.005	0.0050	0.0050	0.0050	0.0050
0.01	0.0100	0.0099	0.0100	0.0099
0.03	0.0280	0.0269	0.0281	0.0269
0.05	0.0379	0.0350	0.0397	0.0349
0.08	0.0347	0.0306	0.0379	0.0306
0.10	0.0292	0.0262	0.0302	0.0261

Plot of AOQ curve for Double Sampling Plan with
$(e_1 = e_2 = 0), (e_1 = 0.005, e_2 = 0), (e_1 = 0, e_2 = 0.05)(e_1 = 0.005, e_2 = 0.001)$

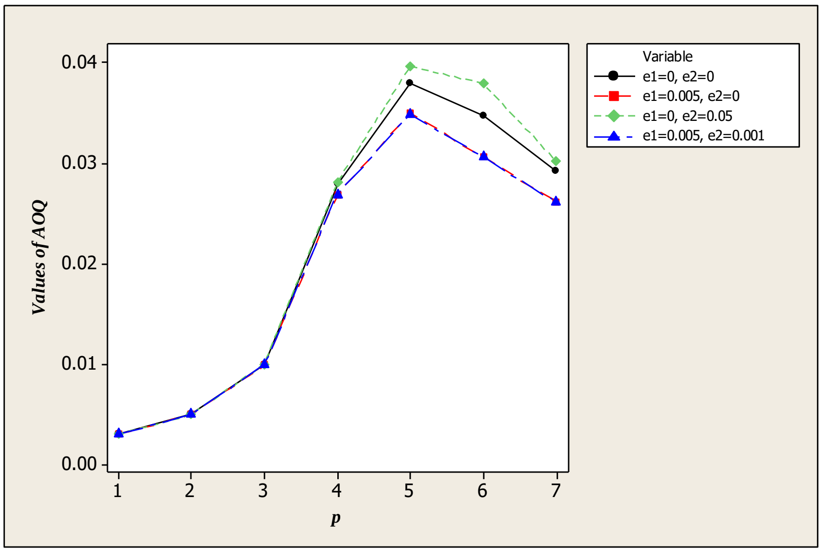

Figure 6: Observe AOQ Curve of Double Sampling Plan for Error prone

Table 7: Values of OC for Semi-Curtailed Single Sampling Plan by Attributes for varying error sizes

$$n = 50, k = 2$$

p \ P_a	$e_1 = 0.0$, $e_2 = 0.0$	$e_1 = 0.005$, $e_2 = 0.0$	$e_1 = 0.0$, $e_2 = 0.25$	$e_1 = 0.005$, $e_2 = 0.25$
0.005	0.9730	0.9109	0.9847	0.9250
0.01	0.9106	0.8282	0.9456	0.8723
0.03	0.5553	0.4761	0.6094	0.6040
0.05	0.2794	0.2333	0.4361	0.3736
0.07	0.1265	0.1089	0.2543	0.2155
0.10	0.0327	0.0677	0.1900	0.1605
0.15	0.0029	0.0023	0.0188	0.0159

ASN function of Semi-Curtailed Single Sampling Plan

Proceeding in a similar way as in the case of OC function, the formula for Observed ASN of semi-curtailed single sampling is derived as

$$ASN_E = (n - \frac{k}{p_e}) p_{a_e} + \frac{n}{p_e} [1 - q_e \binom{n}{k} p_e{}^k q_e{}^{n-k}] \qquad (1.4.8)$$

where p_{a_e} is given by equation (1.3.7) and p_e is given by equation (1.1.1).

Table 8: Error prone ASN for Doubled Sampling Plan by Attributes for varying error sizes
$\mathbf{N = 4000, n_1 = 50, n_2 = 100, c_1 = 0, c_2 = 3}$

p \ ASN	$e_1 = 0.0$, $e_2 = 0.0$	$e_1 = 0.005$, $e_2 = 0.0$	$e_1 = 0.0$, $e_2 = 0.05$	$e_1 = 0.005$, $e_2 = 0.001$
0.003	64	83	63	83
0.005	72	89	69	89
0.01	89	102	88	102
0.03	121	123	121	123
0.05	118	114	120	113
0.08	92	87	95	87
0.10	76	76	79	75

Plot of ASN curves of Double Sampling Plan with
$(e_1 = e_2 = 0), (e_1 = 0.005, e_2 = 0), (e_1 = 0, e_2 = 0.05)(e_1 = 0.005, e_2 = 0.001)$

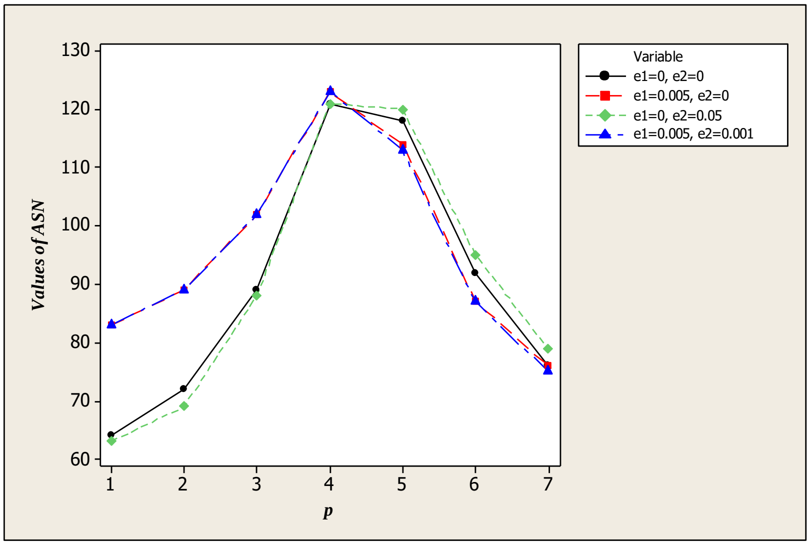

Figure 7: Observe ASN Curve of DSP for Error prone

Table 9: Error prone ASN for Semi-Curtailed Sampling Plan by Attributes for varying error sizes

$$n = 50, k = 2$$

p \ P_a	$e_1 = 0.0$ $e_2 = 0.0$	$e_1 = 0.005$ $e_2 = 0.0$	$e_1 = 0.0$ $e_2 = 0.25$	$e_1 = 0.005$ $e_2 = 0.25$
0.005	49.56	48.45	49.75	48.79
0.01	48.44	46.88	49.07	47.73
0.03	45.00	38.86	44.01	42.06
0.05	32.87	31.17	37.76	35.94
0.07	26.39	25.39	31.97	30.47
0.10	19.61	18.84	25.03	24.02
0.15	13.31	12.95	17.59	17.09

4. CONCLUSION

(i) Observe OC of Single Sampling Plan: When AQL is o.01, then for error free case, the OC value is found to be 0.08 but for error prone case with $e_1 = 0.005$, $e_2 = 0.02$, α value becomes 0.18. So, we can say that the producer risk is increasing. When LTPD = 0.10, with error free case, the consumer risk is 0.003 but with error $e_1 = 0.005$, $e_2 = 0.02$, the consumer risk is 0.0023, which shows consumer risk is decreasing.

(ii) Observe OC of Double Sampling Plan: Similarly, when AQL is o.03, then for error free case, the OC value is found to be 0.0657 but for error prone case with $e_1 = 0.005$, $e_2 = 0.02$, α value becomes 0.1021. Thereby it is saying that the producer risk is increasing. When LTPD = 0.08, with error free case, the consumer risk is 0.4335 but when $e_1 = 0.005$, $e_2 = 0.02$, the consumer risk is 0.3826. Thereby, it is seen that consumer risk is decreasing.

(iii) Observe OC of RGS Plan: Again, when AQL is o.03, then for error free case, the OC value is found to be 0.4546 but for error prone case with $e_1 = 0.005$, $e_2 = 0.02$, α value becomes 0.5759. We can say that the producer risk is increasing. When LTPD = 0.10, with error free case, the consumer risk is 0.0028 but for error prone values of $e_1 = 0.005$, $e_2 = 0.02$, the consumer risk is 0.0023, which shows consumer risk is decreasing.

In general, both for Single and Double Sampling Plan for increasing error sizes the producer risk is increasing but consumer risk is decreasing.

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