FUZZIFICATION OF FALSE POSITION METHOD AND ITS ASSESSMENT

Toralima Bora
Centre for Computer Science and Applications, Dibrugarh University.

Abstract

Fuzzy logic deals with reasoning that is approximate rather than exact. The concept of "fuzzy logic" was introduced in concurrence with the proposal of fuzzy set theory by Lotfi A. Zadeh in the year 1965. Fuzzy logic has been applied to many fields.

Numerical analysis is not about exactness. In case of inexact information, the numerical analyst tries to get a reliable measure of uncertainty which is the outcome of that inexactness and also attempts to acquire an approximation.

Considering these facts, in this paper an attempt has been made to fuzzify one popular numerical method viz., False Position method. In this study, the False Position method has been fuzzified using triangular fuzzy number. The fuzzified form has been used to find the solution of a randomly selected set of problems. The results obtained through the fuzzified form of this method have been compared with the respective classical method. For this fuzzified method, computer program has been developed. The number of iterations required to solve a particular problem by the fuzzified method and the classical method are also compared. Before using the statistical test to compare the fuzzified method and the classical method box-plot is constructed and K-S test is used to check the normality of the data. For comparing the results and the number of iterations, paired-t test has been applied and it has been observed that the False Position method provide approximately same results in both the classical and fuzzified form.

Key words: Fuzzy, False Position method, triangular fuzzy number, box-plot, K-S test, paired-t test.

1. BRIEF INTRODUCTION

Fuzzy logic deals with reasoning that is approximate rather than exact. The concept of "fuzzy logic" was introduced in concurrence with the proposal of fuzzy set theory by Lotfi A. Zadeh in the year 1965. Fuzzy logic has been applied to many fields[1].

Numerical analysis is not about exactness. In case of inexact information, the numerical analyst tries to get a reliable measure of uncertainty which is the outcome of that inexactness and also attempts to acquire an approximation.

Considering these facts, in this paper an attempt has been made to fuzzify one popular numerical method viz., False Position method.

Fuzzification of False Position method has been done using triangular fuzzy number. The left and the right spread of the triangular fuzzy numbers have been taken .01 in the initial stage. But it has been observed that in the intermediate results of calculations and in the final result of calculations the left and right spread of the triangular fuzzy numbers have been changed. The results have been obtained from the computer programs developed for this new fuzzified method. Then the triangular fuzzy number has been defuzzified to compare it with the equivalent crisp number. The results obtained from the newly developed method and the classical methods are compared using statistical tools.

To study the difference in results (solutions) given by the classical method with the fuzzified method a random sample of problems (examples) have been selected. The selection of the examples is done keeping in view the nature, scope and object of the present study. For selecting the problems, at first, three books on numerical methods are selected randomly. From these books, all the problems that can be solved by False Position method are listed. From these lists a sample of 15 examples are selected at random.

2. FUZZIFICATION OF FALSE POSITION METHOD

The fuzzification of False Position method is described below.

Let us consider $F(X) = 0$

Let $F(A)$ and $F(B)$ be of opposite signs and let $A<B$.

Let A=[A_1, A_2, A_3] and B=[B_1, B_2, B_3] . Then there is a root of F(X)=0 lying between A and B. Now fuzzy membership function of A and B are respectively,

$$\mu_A = \begin{cases} \frac{X - A_1}{A_2 - A_1}, A_1 \le X \le A_2 \\ \frac{X - A_3}{A_2 - A_3}, A_2 \le X \le A_3 \\ 0\ , otherwise \end{cases}$$

$$\mu_B = \begin{cases} \frac{X - B_1}{B_2 - B_1}, B_1 \le X \le B_2 \\ \frac{X - B_3}{B_2 - B_3}, B_2 \le X \le B_3 \\ 0\ , otherwise \end{cases}$$

As a first approximation, the root of F(X)=0 is

$$X_1 = \frac{AF(B) - BF(A)}{F(B) - F(A)}$$

Let us consider X_1 =[X_1', X_1'', X_1''']

The membership function of X_1 is

$$\mu_{X_1}(X) = \begin{cases} \frac{X - X_1'}{X_1'' - X_1'}, X_1' \le X \le X_1'' \\ \frac{X - X_1'''}{X_1'' - X_1'''}, X_1'' \le X \le X_1''' \\ 0\ , otherwise \end{cases}$$

Suppose $F(X_1)$ and $F(B)$ are of opposite signs then X_2 lies between X_1 and B

$$X_2 = \frac{X_1 F(B) - BF(X_1)}{F(B) - F(X_1)}$$

Let us consider X_2=[X_2', X_2'', X_2''']

The membership function X_2 is

$$\mu_{X_2}(X) = \begin{cases} \frac{X - X_2'}{X_2'' - X_2'}, X_2' \le X \le X_2'' \\ \frac{X - X_2'''}{X_2'' - X_2'''}, X_2'' \le X \le X_2''' \\ 0 \text{ , } otherwise \end{cases}$$

Similarly we get X_3, X_4 and so on.

This sequence X_1, X_2, X_3, X_4 will converge to the required root.

Example Solve the equation $\mathrm{X}^3 - [2,2,2]\mathrm{X} - [5,5,5] = [0,0,0]$

Solution:

Here $F(X) = \mathrm{X}^3 - [2,2,2]\mathrm{X} - [5,5,5]$

Let A=[1.99, 2, 2.01] and B=[2.99, 3, 3.01] .Since F(A) and F(B) are of opposite signs, therefore there is a root of F(X)=0 lying between A and B. Now fuzzy membership function of A and B are respectively,

$$\mu_A(X) = \begin{cases} \frac{X - 1.99}{2 - 1.99}, 1.99 \le X \le 2 \\ \frac{X - 2.01}{2 - 2.01}, 2 \le X \le 2.01 \\ 0 \text{ , } otherwise \end{cases}$$

$$\mu_B(X) = \begin{cases} \frac{X - 2.99}{3 - 2.99} 2.99, \le X \le 3 \\ \frac{X - 3.01}{3 - 3.01}, 3 \le X \le 3.01 \\ 0 \text{ , } otherwise \end{cases}$$

As a first approximation, the root of F(X)=0 is

$$X_1 = \frac{AF(B) - BF(A)}{F(B) - F(A)}$$

$$= \frac{[1.99,2,2.01]F([2.99,3,3.01]) - [2.99,3,3.01]F([1.99,2,2.01])}{F([2.99,3,3.01]) - F([1.99,2,2.01])}$$

$= 1.94112\ 2.05882\ 2.18308$

Let us consider X_1 =[1.94112, 2.05882, 2.18308]

The membership function of X_1 is

$$\mu_{X_1}(X) = \begin{cases} \frac{X - 1.94112}{2.05882 - 1.94112}, 1.94112 \le X \le 2.05882 \\ \frac{X - 2.18308}{2.05882 - 2.18308}, 2.05882 \le X \le 2.18308 \\ 0\ , otherwise \end{cases}$$

Again $F(X_1)$ and $F(B)$ are of opposite signs therefore the root, say X_2 lies between X_1 and B

$$X_2 = \frac{X_1 F(B) - BF(X_1)}{F(B) - F(X_1)}$$

$= \frac{[1.94112,2.05882,2.18308]F([2.99,3,3.01])-[2.99,3,3.01]F([1.94112,2.05882,2.18308])}{F([2.99,3,3.01])-F([1.94112,2.05882,2.18308])}$

= [1.41283 2.08126, 2.94182]

The membership function of X_2 is

$$\mu_{X_2}(X) = \begin{cases} \frac{X - 1.41283}{2.08126 - 1.41283}, 1.41283 \le X \le 2.08126 \\ \frac{X - 2.94182}{2.08126 - 2.94182}, 2.08126 \le X \le 2.94182 \\ 0\ , otherwise \end{cases}$$

Similarly we get X_3, X_4 and so on.

3. FUZZIFIED FALSE POSITION METHOD VS CLASSICAL FALSE POSITION METHOD

To compare the results of Fuzzified False Position Method and Classical False Position Method presented in Table 1, statistical test has been used. C++ programs have been developed for Fuzzified False Position Method and Classical False Position Method. The Table 1 shows the solution of the examples obtained from Fuzzified False Position Method and Classical False Position Method. Solutions of the examples have been computed by the program developed for Fuzzified False Position Method and the program developed for Classical False Position Method. The defuzzified values of the triangular fuzzy numbers have been compared with the corresponding crisp values of the classical method.

The following hypotheses have been constructed for comparing the results.

Table 1: Showing the Output of the C++ Program Developed for the Fuzzified False Position Method and Classical False Position Method

Sl no	Iteration (fuzzy)	Root (Triangular fuzzy no)	Iteration (classical)	Crisp root	Defuzzified value
1	10	-3.07400 , 2.71541, 3.06365	9	2.7065	2.70653
2	14	-5.42649, 2.94272, 5.4854	17	2.9428	2.94282
3	7	-0.73306, 0.73206, 1.57085	10	0.7321	0.732055
4	11	-5.14549, 2.0946, 4.31413	13	2.0946	2.09455
5	9	-5.91660, 2.37155, 3.98500	11	2.3736	2.37365
6	14	-0.25800, 0.56713, 1.35953	13	0.56714	0.567143
7	8	-0.21600, 0.56703, 1.4676	7	0.5671	0.567143
8	8	-.28706, .85250, 1.607261	8	0.85261	0.852605
9	18	-0.74313, 1.4882, 3.43850	17	1.04991	1.04991
10	7	-0.24686, 0.3573, 0.46862	8	0.3574	0.357403
11	6	-2.66224, .73917, 2.96213	7	0.7391	0.739085
12	4	-3.4631, 3.7990, 9.92523	5	3.7893	3.78928
13	13	-1.95688, 2.7902, 6.18584	6	2.7984	2.79839
14	19	-2.29013, -1.2480, 2.57977	18	-1.2491	-1.24905
15	6	-2.47928, 0.6070, 2.15651	7	0.6071	0.607102

Comparison of the Solutions obtained from Fuzzified False Position Method and Classical False Position Method:

In this section, statistical test has been employed to make comparison between the the solutions obtained from the classical and fuzzified False Position method.

The null hypothesis for comparing the results is as follows,

H_{01} : The results obtained by Fuzzified False Position method and Classical False Position methods are same.

Against the two-sided alternative hypothesis

H_{A1}: The results obtained by Fuzzified False Position method and Classical False Position methods are not same.

Before applying the statistical test, the normality of the data has been checked by using box-plot and K-S test. The box-plots of the results obtained by classical and fuzzified form of the False Position Method are presented in Fig.1,

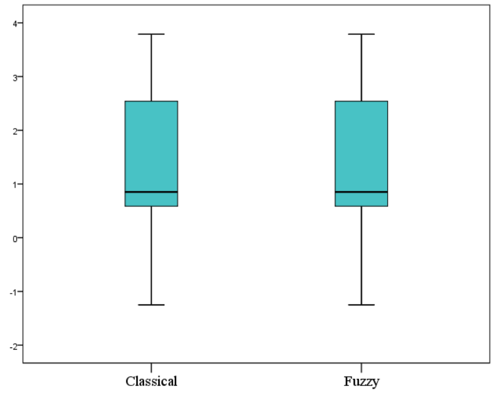

Figure 1: Box-plot of False Position Method

By observing the position of median value it is likely that the data do not follow normal distribution. For confirmation, the K-S test has been conducted considering the null hypothesis H_0: data follows normal distribution Vs H_A: the data do not follow normal distribution. The results of the K-S test are presented in the Table 2.

Table 2: Results of K-S test for the Solutions of Classical False Position Method and Fuzzified False Position Method

	Kolmogorov-Smirnov		
	Statistic	d.f.	p-value
Fuzzified form	0.190	16	0.127
Classical form	0.190	16	0.127

From the Table 2 it can be observed that the results obtained by both classical False Position Method and fuzzified False Position Method to a sample of problems may follow normal distribution (p-value >0.05, H_0 may be accepted). Thus to compare the results parametric test i.e., paired-t test has been applied. The results of paired-t test together with different descriptive statistics are presented in Table 3.

Table 3: Showing the Results of Descriptive Statistics and Paired-t test for the Solutions of Fuzzified False Position Method and Classical False Position Method

Measures	Classical	Fuzzified	t-value	p-value
Mean	1.4793	1.4793		
Median	0.9513	0.9513		
Std. Deviation	1.3242	1.3243		
Minimum	-1.2491	-1.2491		
Maximum	3.7893	3.7893	1.464	0.164

By observing the results presented in Table 3, we may accept H_0 as the p-value is >0.05 Thus, it can be concluded that the classical False Position Method and the fuzzified form of the False Position Method give same result to the same problem.

Comparison of the number of Iteration Required by both the classical and fuzzified form of False Position Method:

To make comparison of the number of iteration required by the classical and fuzzified form of False Position Method to solve same set of problems, statistical test has been employed.

The null hypothesis for comparing the iteration required is as follows,

H_{02} : The Iteration numbers Required by Fuzzified False Position Method and Classical False Position Method are Same.

Against the two-sided alternative hypothesis

H_{A2}: The Iteration numbers Required by Fuzzified False Position Method and Classical False Position Method are not Same.

Before applying the statistical test, the normality of the data has been checked by using box-plot and K-S test. The box-plots of the number of iteration required by classical and fuzzified form of the False Position Method are presented in Fig.2.

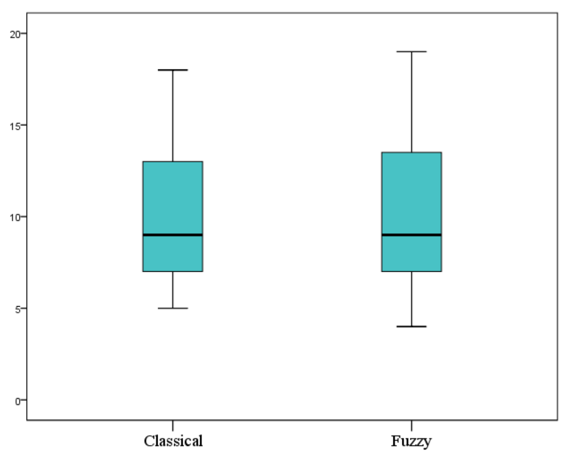

Figure 2: Box-plot of False Position Method

By observing the position of median value it is likely that the data do not follow normal distribution. For confirmation, the K-S test has been conducted considering the null hypothesis H_0: data follows normal distribution Vs H_A: the data do not follow normal distribution. The results of the K-S test are presented in the Table 4.

Table 4: Results of K-S test for the Iterations of Classical False Position Method and Fuzzified False Position Method

	Kolmogorov-Smirnov		
	Statistic	d.f.	p-value
Classical	.180	15	.200*
Fuzzy	.160	15	.200*

From the Table 4 it can be observed that the number of iteration required by both classical False Position Method and fuzzified False Position Method to solve problems may follow normal distribution (p-value >0.05, H_0 may be accepted). Thus to compare the results parametric test i.e., paired-t test is applied. The results of paired-t test together with different descriptive statistics are presented in Table 5.

Table 5: Showing the Results of Descriptive Statistics and Paired-t test for the Iterations of Fuzzified False Position Method and Classical False Position Method

	Classical	Fuzzy	t-value	p-value
Mean	10.4000	10.2667		
Median	9.0000	9.0000		
Std. Deviation	4.27284	4.47958		
Minimum	5.00	4.00		
Maximum	18.00	19.00	0.211	0.836

By observing the results presented in Table 5, we may accept H_0 as the p-value is >0.05. Thus, it can be concluded that the number iteration required by the classical False Position Method and the fuzzified form of the False Position Method to solve the same problem are more or less same.

4. CONCLUSION

With the help of descriptive and inferential statistical tool, various tests have been performed and it has been seen that the False Position method provide more or less same results in classical and fuzzified form.

In the same way, the number of iterations required by the classical and fuzzified False Position methods has been compared using appropriate statistical tool. It has been observed that iteration number required by the classical method is almost same with the iteration number required by fuzzified method.

REFERENCES

[1] Zadeh, A. L. (1965). Fuzzy Set, *Information and Control*, 8(3), 338–353.