DETERMINATION OF STEADY STATE SOLUTIONS OF GROUND WATER LEVELS IN ANANTAPURAMU DISTRICT THROUGH MARKOV CHAIN MODEL

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Abstract

Background: Markov Chain (M.C) Analysis is an advanced tool of data analysis used in many real life problems like Meteorology, Agriculture, Industry, and Reliability Analysis of equipments and so on.

Method: In this paper we have considered the Markov Chain Analysis of Ground Water Levels in Anantapuramu District of Andhra Pradesh. In Markov Chain Analysis we define different states of the Markov Chain and consider Transitions occurred from one state to another state. Which are popularly known as Transition of the Markov Chain using these Transitions, we formulate Transition Probability Matrix (T.P.M) which helps us to calculate Limiting Probabilities and Limiting Probabilities of each state. With the help of these probabilities one can determine whether the states of the Markov Chains are ergodic, transient or recurrent. Thus we can obtain Steady State Solutions of Markov Chain.

Result: It deals with the application of Markov model to analyze the Ground water levels (GWLs) in Anantapuramu district based on the data collected from January 2001 to November 2017.

Conclusion: For development of Markov model, for the purpose of analysis, the district is divided into five Zones(Revenue Divisions) and conclusions are drawn based on the results obtained, for each zone.

Key words: Markov model, Ground Water Level, Transient, Recurrentand Steady State Solutions.

1. INTRODUCTION

Earlier we have started the work by collecting the data on Rainfall (R.F) and Ground Water Levels (GWLs) in Anantapuramu district. The RFdata is collected from Chief Planning Office Anantapuramu and The GWLs data is collected from Ground Water and Water Audit Department Anantapuramu since 2001 January to 2017 November. For the collected data have applied Time Series Analysis on RF and GWLs data (Raju and Mohammed, 2019a), Modified Exponential Model (Raju and Mohammed, 2019b), Gompertz Model (Raju and Mohammed, 2019c) and Logistic Model (Raju and Mohammed, 2019d) and conclusions are drawn based on the results obtained. Now we proceed to apply, advanced Statistical tool in Stochastic Processes i.e. Markov Chain Model, only we will take the GWL Data in this paper.

Definition of Markov Chain: "A Stochastic model describing a sequence of possible events in which the probability of each event depend only the state attained in the previous event".

Markov chain is a special case of Stochastic Process where state space (Ω) and index set (T) are both discrete variables. In general Stochastic Process is defined as a function of two arguments namely state space and index set. Usually state space is denoted as 'Ω' and index set by 'T'. If 'T' is discrete set of time points$t_1, t_2, t_3, \ldots, t_n$ we obtain time series. In Markov chain model along with the time we assume the state space 'Ω' is also discrete Medhi (1994). Since the main variable in this paper is Ground water level of Anantapuramu district we defined three state Markov chain namely {-1, 0, +1} where '-1' represents increasing the depth of the Ground water level from a higher level from the ground. '0' represents no change in the Ground water level from the previous period and '+1' represents the depth of the Ground water level decreasing from a lower depth. i.e. water level is increased from a lower level.

Consider the above three states of a Markov chain the collected data on Ground water level is transformed into three stages and Transition frequencies are formulated through which Transition probabilities are calculated in the following section.

After forming the Transition Probability Matrix for different zones steady state solutions are obtained for the behavior of Ground water levels for the forth coming years. For the purpose of analysis The Anantapuramu district is divided into Five Zones as explained in the following Table 1.

Table 1: Zonal wise (Revenue Division) classification of Anantapuramu District.

Zones	**Zone –I**	**Zone –II**	**Zone –III**	**Zone –IV**	**Zone –V**
Name of the Revenue Division (RD)	Anantapuramu RD	Penukonda RD	Kadiri RD	Kalyandurg RD	Dharmavaram RD
Piezometer Points (194)	54	50	31	32	27

2. BODY PART

Markov chain analysis:

From the collected data on Ground water levels following tables for different zones representing monthly averages from 2001 (Jan) to 2017 (Nov) are calculated and are given in the following tables.

Table 2: Monthly averages of Ground water levels in Zone-I of Anantapuramu district.

Months / Years	**Jan**	**Feb**	**Mar**	**April**	**May**	**June**	**July**	**Aug**	**Sep**	**Oct**	**Nov**	**Dec**
2001	8.15	8.35	8.55	8.94	9.63	10.33	10.77	11.08	9.92	4.77	4.83	4.89
2002	5.16	6.11	7.09	7.16	8.50	7.54	8.03	8.71	9.69	9.36	9.16	9.52
2003	12.16	13.97	12.85	12.56	13.40	13.76	14.08	14.31	15.36	12.92	13.76	13.70
2004	13.42	14.44	15.36	16.00	15.08	14.58	13.36	13.92	13.80	13.89	13.81	13.42
2005	13.70	14.08	14.64	15.34	15.97	16.08	16.20	14.81	12.18	9.31	9.31	8.86
2006	8.87	9.52	10.12	9.94	11.02	11.32	11.60	12.79	12.10	11.86	11.32	12.04
2007	12.27	12.18	12.82	13.40	13.91	12.48	10.68	11.09	7.58	6.67	7.21	6.97
2008	8.08	8.34	9.88	10.23	10.12	10.44	10.67	11.65	10.09	10.28	10.19	9.65
2009	9.17	10.45	11.09	11.78	12.32	12.67	14.23	14.92	13.65	12.35	12.04	11.44
2010	11.24	12.40	12.92	14.30	15.49	14.54	13.40	13.40	12.61	11.85	10.55	10.22
2011	11.27	11.78	12.10	12.51	12.52	12.85	12.88	13.11	13.05	12.52	13.43	14.29
2012	12.78	14.47	15.73	15.58	16.75	18.09	16.84	16.89	14.48	13.49	12.84	12.28
2013	13.25	14.16	14.90	16.66	17.19	17.47	18.04	18.62	16.92	14.12	14.71	15.50
2014	15.36	15.19	15.57	16.59	15.66	17.24	17.63	15.57	15.82	16.45	15.19	15.87
2015	14.94	15.90	15.85	16.02	16.37	16.44	16.72	15.86	14.34	12.28	11.77	12.14
2016	13.11	14.15	15.70	16.73	16.99	16.52	14.96	15.69	15.60	15.64	15.97	15.82
2017	16.42	16.99	18.02	18.90	19.63	18.86	19.54	20.24	16.96	11.40	9.94	-----

Table 3: Monthly averages of Ground water levels in Zone-II of Anantapuramu district.

Months / Years	Jan	Feb	Mar	April	May	June	July	Aug	Sep	Oct	Nov	Dec
2001	13.78	13.98	14.55	14.53	15.14	15.27	16.82	17.41	16.68	11.42	10.29	10.36
2002	10.39	10.85	11.80	12.56	13.23	13.47	14.09	14.61	15.99	16.48	16.67	18.14
2003	12.67	13.39	20.76	21.09	21.73	22.57	23.35	23.28	23.81	23.85	20.05	20.11
2004	20.70	21.77	23.01	23.36	22.25	22.26	22.56	22.67	22.07	22.53	23.04	24.60
2005	25.02	25.66	27.24	27.96	28.26	29.47	29.59	26.28	21.24	17.30	17.30	15.58
2006	16.26	17.39	18.18	19.55	19.52	20.49	19.98	20.70	21.63	21.24	21.35	19.73
2007	22.16	23.77	24.75	25.7	25.89	24.62	24.77	23.97	22.17	18.33	17.28	17.61
2008	21.90	21.84	22.95	22.69	23.49	23.51	23.73	22.97	18.12	16.22	15.82	15.46
2009	15.89	17.15	17.50	18.07	18.75	18.55	19.56	20.25	18.23	15.35	15.38	15.66
2010	15.40	16.33	15.91	16.29	16.36	15.67	15.88	15.44	15.64	14.80	11.44	11.66
2011	12.08	13.36	13.93	14.67	15.45	15.54	16.45	16.54	15.48	15.74	16.03	16.72
2012	18.51	19.73	19.35	19.61	20.84	21.40	21.80	22.37	21.44	20.15	19.82	20.98
2013	23.09	21.49	22.89	24.90	26.85	28.40	28.01	24.71	19.90	18.71	18.53	19.60
2014	19.98	20.26	20.70	24.28	20.73	23.13	24.48	25.73	25.97	25.19	24.83	25.78
2015	27.62	28.79	28.27	29.11	29.09	28.39	29.48	30.06	26.83	24.11	20.87	18.88
2016	19.99	21.95	24.27	26.37	26.18	25.14	27.34	25.15	27.86	31.83	33.65	33.65
2017	36.12	36.92	40.02	41.63	42.01	41.68	43.23	44.76	43.23	34.83	26.54	-----

Table 4: Monthly averages of Ground water levels in Zone-III of Anantapuramu district.

Months / Years	Jan	Feb	Mar	April	May	June	July	Aug	Sep	Oct	Nov	Dec
2001	10.41	10.61	10.90	11.20	12.28	11.71	13.17	12.71	12.67	5.75	5.10	5.80
2002	6.17	6.74	7.99	8.21	9.04	8.88	9.56	10.25	11.08	10.35	10.37	10.80
2003	14.61	15.73	13.39	13.85	14.40	15.19	15.91	15.89	16.85	13.15	12.46	12.57
2004	13.63	14.66	15.67	15.96	14.72	15.58	15.76	16.37	16.60	15.14	16.13	17.24
2005	17.75	18.29	18.76	19.53	20.78	21.09	21.32	18.66	15.05	8.22	8.22	7.46
2006	7.95	8.75	9.42	10.23	10.85	10.89	11.40	12.16	12.72	12.87	12.91	10.87
2007	13.84	14.56	15.50	16.48	16.60	15.82	15.51	14.72	13.18	12.05	10.91	11.61
2008	8.80	9.53	9.51	9.81	10.53	10.35	11.63	11.49	8.30	7.19	7.36	6.90
2009	8.67	9.95	10.82	11.65	12.21	10.24	12.15	13.29	12.30	10.78	10.80	10.15
2010	11.05	11.58	13.46	14.25	14.58	14.46	13.34	13.34	12.94	12.68	6.34	6.34
2011	7.49	9.20	10.71	11.76	12.43	13.12	14.11	12.57	11.58	12.43	11.53	10.74
2012	13.07	14.36	15.69	15.95	15.95	16.82	17.46	17.02	17.65	17.08	16.26	16.62
2013	18.14	20.05	18.52	23.50	24.81	24.75	25.22	22.56	17.28	12.14	10.97	11.16
2014	12.49	13.80	14.84	14.63	18.51	17.93	20.93	23.71	24.96	25.44	24.28	25.63
2015	27.85	28.67	31.74	32.17	33.50	27.56	27.80	30.97	27.81	25.07	14.27	9.24
2016	9.73	11.80	12.59	13.54	13.94	13.89	14.43	13.89	15.41	19.94	23.78	21.25
2017	22.70	26.68	26.33	28.86	29.15	31.2	31.20	32.70	22.58	17.26	15.66	-----

Table 5: Monthly averages of Ground water levels in Zone-IV of Anantapuramu district.

Months / Years	Jan	Feb	Mar	April	May	June	July	Aug	Sep	Oct	Nov	Dec
2001	9.63	9.83	10.04	11.66	12.42	12.11	12.92	13.08	12.97	7.41	11.31	7.87
2002	8.12	8.80	9.70	10.35	10.44	10.45	10.78	10.83	11.24	10.90	11.11	11.45
2003	11.94	12.53	13.19	13.84	14.69	14.99	15.85	15.96	16.17	16.35	15.86	15.72
2004	16.27	16.85	17.68	18.02	17.17	16.81	17.21	17.41	17.51	17.97	17.65	19.30
2005	19.87	19.89	20.54	21.06	21.47	21.33	20.56	18.16	15.15	11.86	11.86	11.65
2006	12.51	13.20	13.26	14.60	14.08	14.75	15.28	15.92	15.94	15.82	15.14	13.90
2007	14.72	15.13	16.43	16.80	17.46	16.10	15.89	16.43	14.50	13.08	11.44	12.03
2008	11.72	11.85	12.24	12.26	12.67	11.89	12.30	12.25	8.41	8.08	8.36	8.50
2009	8.93	9.68	10.17	10.85	11.03	10.78	11.29	11.49	10.11	7.19	6.56	6.82
2010	8.24	8.22	9.09	9.70	10.42	9.76	9.61	9.01	8.47	7.71	6.09	6.60
2011	6.74	7.09	7.83	8.32	8.47	8.67	9.28	9.67	10.07	10.15	9.92	11.15
2012	11.69	12.85	13.98	13.67	13.68	14.91	15.03	14.49	14.56	13.46	13.14	13.70
2013	14.68	15.64	16.72	17.57	17.98	18.51	18.44	18.89	16.90	15.70	15.82	16.78
2014	16.46	18.74	18.53	19.28	19.55	20.38	19.71	19.18	19.10	18.82	18.43	18.73
2015	19.37	19.76	20.65	21.15	21.04	20.75	20.77	21.65	20.07	16.08	14.83	14.76
2016	15.93	16.88	18.19	18.87	19.09	18.16	19.95	20.37	20.47	21.23	22.05	22.73
2017	23.16	23.51	23.78	25.27	26.33	26.45	27.02	27.63	27.25	23.28	19.03	-----

Table 6: Monthly averages of Ground water levels in Zone-V of Anantapuramu district.

Months / Years	Jan	Feb	Mar	April	May	June	July	Aug	Sep	Oct	Nov	Dec
2001	11.32	11.52	11.96	12.29	13.05	13.67	14.71	15.53	14.49	5.12	4.81	6.18
2002	6.43	7.11	8.60	9.26	8.17	7.92	9.36	9.54	10.43	9.98	10.35	10.87
2003	14.29	15.64	13.20	13.89	14.54	15.40	16.29	16.17	16.19	15.78	14.23	14.40
2004	15.34	16.00	16.67	17.26	16.75	16.99	16.30	16.45	17.37	16.92	17.39	18.33
2005	18.44	19.12	20.24	20.83	22.51	21.94	22.11	19.55	14.80	12.67	12.67	12.49
2006	13.74	14.77	15.63	15.20	15.52	16.98	15.79	16.29	16.67	17.49	18.17	21.32
2007	17.11	17.82	18.97	20.11	20.99	19.37	18.53	18.48	15.60	13.29	11.88	12.17
2008	8.41	8.67	9.39	10.32	11.11	10.88	11.41	11.19	7.36	6.47	7.26	6.57
2009	8.02	8.08	8.25	8.88	9.48	9.42	10.65	12.58	12.32	11.79	11.93	11.54
2010	11.04	11.64	12.73	13.69	13.34	12.10	11.83	11.76	11.70	12.14	9.36	10.16
2011	10.52	11.52	12.22	12.91	12.87	13.33	13.84	13.18	13.58	13.33	12.74	13.72
2012	10.74	12.03	12.65	13.40	13.37	13.12	13.72	13.80	14.70	13.62	13.75	13.45
2013	15.27	15.21	16.73	15.46	16.56	16.48	16.84	14.31	10.85	10.63	10.91	11.52
2014	12.53	11.57	13.04	12.71	15.70	14.07	14.67	17.48	18.58	18.44	16.99	18.71
2015	19.15	19.44	18.98	20.15	19.84	16.24	19.04	17.53	17.44	16.42	14.25	13.61
2016	13.83	14.39	16.84	17.35	17.90	15.09	14.36	14.59	15.47	16.35	18.75	18.71
2017	18.39	19.84	19.53	21.20	20.80	20.48	21.08	23.11	20.32	12.65	8.72	-----

From the above tables Transitions are formed fromi^{th} month to$(i+1)^{th}$ month. i.e. Jan-2001 to Nov-2017 and are given in the following tables. In the following tables Markov states -1, 0, +1 are considered as follows.

'-1' indicates "Depth of the Ground water level increasing".

'0' indicates "No change in the Ground water level".

'+1' indicates "Depth of the Ground water level decreasing".

Table 7: Transitions and Frequencies for Zone-I of Anantapuramudistrict (Using Table 2).

Years / Months	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec	-1To-1	-1To0	-1To+1	0To-1	0To0	0To+1	+1To-1	+1To0	+1To+1
2001	---	-1	-1	-1	-1	-1	-1	-1	+1	+1	-1	-1	7	0	1	0	0	0	1	0	1
2002	-1	-1	-1	-1	-1	+1	-1	-1	-1	+1	+1	-1	7	0	2	0	0	0	2	0	1
2003	-1	-1	+1	+1	-1	-1	-1	-1	-1	+1	-1	+1	6	0	3	0	0	0	2	0	1
2004	+1	-1	-1	-1	+1	+1	+1	-1	+1	-1	+1	+1	2	0	3	0	0	0	3	0	4
2005	-1	-1	-1	-1	-1	-1	-1	+1	+1	+1	0	+1	6	0	1	0	0	1	1	1	2
2006	-1	-1	-1	+1	-1	-1	-1	-1	+1	+1	+1	-1	5	0	2	0	0	0	3	0	2
2007	-1	+1	-1	-1	-1	+1	+1	-1	+1	+1	-1	+1	3	0	4	0	0	0	3	0	2
2008	-1	-1	-1	-1	+1	-1	-1	-1	+1	-1	+1	+1	5	0	3	0	0	0	3	0	1
2009	+1	-1	-1	-1	-1	-1	-1	-1	+1	+1	+1	+1	6	0	1	0	0	0	1	0	4
2010	+1	-1	-1	-1	-1	+1	+1	0	+1	+1	+1	+1	3	0	1	0	0	1	1	1	5
2011	-1	-1	-1	-1	-1	-1	-1	-1	+1	+1	-1	-1	8	0	1	0	0	0	2	0	1
2012	+1	-1	-1	+1	-1	-1	+1	-1	+1	+1	+1	+1	2	0	4	0	0	0	3	0	3
2013	-1	-1	-1	-1	-1	-1	-1	-1	+1	+1	-1	-1	8	0	1	0	0	0	2	0	1
2014	+1	+1	-1	-1	+1	-1	-1	+1	-1	-1	+1	-1	3	0	4	0	0	0	4	0	1
2015	+1	-1	+1	-1	-1	-1	-1	+1	+1	+1	+1	-1	3	0	3	0	0	0	3	0	3
2016	-1	-1	-1	-1	-1	+1	+1	-1	+1	-1	-1	+1	6	0	3	0	0	0	2	0	1
2017	-1	-1	-1	-1	-1	+1	-1	-1	+1	+1	+1	---	5	0	2	0	0	0	2	0	2
												Sum	**85**	**0**	**39**	**0**	**0**	**2**	**38**	**2**	**35**

Table 8: Transitions and Frequencies for Zone-II of Anantapuramu district (Using Table 3).

Months Years	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec	-1To-1	-1To0	-1To+1	0To-1	0To0	0To+1	+1To-1	+1To0	+1To+1
2001	---	-1	-1	+1	-1	-1	-1	-1	+1	+1	+1	-1	4	0	2	0	0	0	2	0	2
2002	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	12	0	0	0	0	0		0	0
2003	+1	-1	-1	-1	-1	-1	-1	+1	-1	-1	+1	-1	6	0	3	0	0	0	3	0	0
2004	-1	-1	-1	-1	+1	-1	-1	-1	+1	-1	-1	-1	7	0	2	0	0	0	2	0	0
2005	-1	-1	-1	-1	-1	-1	-1	+1	+1	+1	0	+1	7	0	1	0	0	1		1	2
2006	-1	-1	-1	-1	+1	-1	+1	-1	-1	+1	-1	+1	4	0	4	0	0	0	4	0	0
2007	-1	-1	-1	-1	-1	+1	-1	+1	+1	+1	+1	-1	4	0	2	0	0	0	3	0	3
2008	-1	+1	-1	+1	-1	-1	-1	+1	+1	+1	+1	+1	3	0	3	0	0	0	3	0	4
2009	-1	-1	-1	-1	-1	+1	-1	-1	+1	+1	-1	-1	6	0	2	0	0	0	3	0	1
2010	+1	-1	+1	-1	-1	+1	-1	+1	-1	+1	+1	-1	1	0	5	0	0	0	5	0	1
2011	-1	-1	-1	-1	-1	-1	-1	-1	+1	-1	-1	-1	10	0	1	0	0	0	1	0	0
2012	-1	-1	+1	-1	-1	-1	-1	-1	-1	+1	+1	-1	7	0	2	0	0	0	2	0	1
2013	-1	+1	-1	-1	-1	-1	+1	+1	+1	+1	+1	-1	4	0	2	0	0	0	2	0	4
2014	-1	-1	-1	-1	+1	-1	-1	-1	-1	+1	+1	-1	7	0	2	0	0	0	2	0	1
2015	-1	-1	+1	-1	+1	+1	-1	-1	+1	+1	+1	+1	3	0	3	0	0	0	2	0	4
2016	-1	-1	-1	-1	+1	+1	-1	+1	-1	-1	-1	0	5	1	2	0	0	0	3	0	1
2017	-1	-1	-1	-1	-1	+1	-1	-1	+1	+1	+1	---	5	0	2	1	0	0	1	0	2
											Sum		**95**	**1**	**38**	**1**	**0**	**1**	**38**	**1**	**26**

Table 9: Transitions and Frequencies for Zone-III of Anantapuramu district (Using Table 4).

Months Years	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec	-1To-1	-1To0	-1To+1	0To-1	0To0	0To+1	+1To-1	+1To0	+1To+1
2001	---	-1	-1	-1	-1	+1	-1	+1	+1	+1	+1	-1	3	0	2	0	0	0	2	0	3
2002	-1	-1	-1	-1	-1	+1	-1	-1	-1	+1	-1	-1	8	0	2	0	0	0	2	0	0
2003	-1	-1	+1	-1	-1	-1	-1	+1	-1	+1	+1	-1	5	0	3	0	0	0	3	0	1
2004	-1	-1	-1	-1	+1	-1	-1	-1	-1	+1	-1	-1	8	0	2	0	0	0	2	0	0
2005	-1	-1	-1	-1	-1	-1	-1	+1	+1	+1	0	+1	7	0	1	0	0	1	0	1	2
2006	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	+1	10	0	1	0	0	0	1	0	0
2007	-1	-1	-1	-1	-1	+1	+1	+1	+1	+1	+1	-1	4	0	1	0	0	0	2	0	5
2008	+1	-1	+1	-1	-1	+1	-1	+1	+1	+1	-1	+1	1	0	5	0	0	0	4	0	2
2009	-1	-1	-1	-1	-1	+1	-1	-1	+1	+1	-1	+1	5	0	3	0	0	0	3	0	1
2010	-1	-1	-1	-1	-1	+1	+1	0	+1	+1	+1	0	4	0	1	0	0	1	1	2	3
2011	-1	-1	-1	-1	-1	-1	-1	+1	+1	-1	+1	+1	6	0	2	1	0	0	1	0	2
2012	-1	-1	-1	-1	0	-1	-1	+1	-1	+1	+1	-1	4	1	2	1	0	0	3	0	1
2013	-1	-1	+1	-1	-1	+1	-1	+1	+1	+1	+1	-1	3	0	3	0	0	0	3	0	3
2014	-1	-1	-1	+1	-1	+1	-1	-1	-1	-1	+1	-1	6	0	3	0	0	0	3	0	0
2015	-1	-1	-1	-1	-1	+1	-1	-1	+1	+1	+1	+1	6	0	2	0	0	0	1	0	3
2016	-1	-1	-1	-1	-1	+1	-1	+1	-1	-1	-1	+1	6	0	3	0	0	0	3	0	0
2017	-1	-1	+1	-1	-1	-1	0	-1	+1	+1	+1	---	3	1	2	1	0	0	2	0	2
												Sum	**89**	**2**	**38**	**3**	**0**	**2**	**36**	**3**	**28**

Table 10: Transitions and Frequencies for Zone-IV of Anantapuramu district (Using Table5).

Months / Years	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec	-1To-1	-1To0	-1To+1	0To-1	0To0	0To+1	+1To-1	+1To0	+1To+1
2001	---	-1	-1	-1	-1	+1	-1	-1	+1	+1	-1	+1	4	0	3	0	0	0	2	0	1
2002	-1	-1	-1	-1	-1	-1	-1	-1	-1	+1	-1	-1	9	0	1	0	0	0	2	0	0
2003	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	+1	+1	10	0	1	0	0	0	0	0	1
2004	-1	-1	-1	-1	+1	+1	-1	-1	-1	-1	+1	-1	6	0	2	0	0	0	3	0	1
2005	-1	-1	-1	-1	-1	+1	+1	+1	+1	+1	0	+1	5	0	1	0	0	1	0	1	4
2006	-1	-1	-1	-1	+1	-1	-1	-1	-1	+1	+1	+1	6	0	2	0	0	0	2	0	2
2007	-1	-1	-1	-1	-1	+1	+1	-1	+1	+1	+1	-1	4	0	2	0	0	0	3	0	3
2008	+1	-1	-1	-1	-1	+1	-1	+1	+1	+1	-1	-1	4	0	3	0	0	0	3	0	2
2009	-1	-1	-1	-1	-1	+1	-1	-1	+1	+1	+1	-1	6	0	2	0	0	0	2	0	2
2010	-1	+1	-1	-1	-1	+1	+1	+1	+1	+1	+1	-1	3	0	2	0	0	0	2	0	5
2011	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	+1	-1	10	0	1	0	0	0	1	0	0
2012	-1	-1	-1	+1	-1	-1	-1	+1	-1	+1	+1	-1	5	0	3	0	0	0	3	0	1
2013	-1	-1	-1	-1	-1	-1	+1	-1	+1	+1	-1	-1	7	0	2	0	0	0	2	0	1
2014	+1	-1	+1	-1	-1	-1	+1	+1	+1	+1	+1	-1	2	0	3	0	0	0	3	0	4
2015	-1	-1	-1	-1	+1	+1	-1	-1	+1	+1	+1	+1	5	0	2	0	0	0	1	0	4
2016	-1	-1	-1	-1	-1	+1	-1	-1	-1	-1	-1	-1	9	0	1	0	0	0	2	0	0
2017	-1	-1	-1	-1	-1	-1	-1	-1	+1	+1	+1	---	8	0	1	0	0	0	0	0	2
												Sum	**103**	**0**	**32**	**0**	**0**	**1**	**31**	**1**	**33**

Table 11: Transitions and Frequencies for Zone-V of Anantapuramu district (Using Table 6).

Months / Years	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec	-1To-1	-1To0	-1To+1	0To-1	0To0	0To+1	+1To-1	+1To0	+1To+1
2001	---	-1	-1	-1	-1	-1	-1	-1	+1	+1	+1	-1	6	0	1	0	0	0	1	0	2
2002	-1	-1	-1	-1	+1	+1	-1	-1	-1	+1	-1	-1	7	0	2	0	0	0	2	0	1
2003	-1	-1	+1	-1	-1	-1	-1	+1	-1	+1	+1	-1	5	0	3	0	0	0	3	0	1
2004	-1	-1	-1	-1	+1	-1	+1	-1	-1	+1	-1	-1	6	0	3	0	0	0	3	0	0
2005	-1	-1	-1	-1	-1	+1	-1	+1	+1	+1	0	+1	5	0	2	0	0	1	1	1	2
2006	-1	-1	-1	+1	-1	-1	+1	-1	-1	-1	-1	-1	7	0	2	0	0	0	3	0	0
2007	+1	-1	-1	-1	-1	+1	+1	+1	+1	+1	+1	-1	3	0	2	0	0	0	2	0	5
2008	+1	-1	-1	-1	-1	+1	-1	+1	+1	+1	-1	+1	3	0	4	0	0	0	3	0	2
2009	-1	-1	-1	-1	-1	+1	-1	-1	+1	+1	+1	+1	5	0	2	0	0	0	2	0	3
2010	+1	-1	-1	-1	+1	+1	+1	+1	+1	-1	+1	-1	2	0	2	0	0	0	3	0	5
2011	-1	-1	-1	-1	+1	-1	-1	+1	-1	+1	+1	-1	5	0	3	0	0	0	3	0	1
2012	-1	-1	-1	-1	+1	+1	-1	-1	-1	+1	-1	+1	6	0	3	0	0	0	2	0	1
2013	-1	+1	-1	+1	-1	+1	-1	+1	+1	+1	-1	-1	1	0	4	0	0	0	5	0	2
2014	-1	+1	-1	+1	-1	+1	-1	-1	-1	+1	+1	-1	3	0	4	0	0	0	4	0	1
2015	-1	-1	+1	-1	+1	+1	-1	+1	+1	+1	+1	+1	2	0	3	0	0	0	2	0	5
2016	-1	-1	-1	-1	-1	+1	+1	-1	-1	-1	-1	+1	7	0	2	0	0	0	2	0	1
2017	+1	-1	+1	-1	+1	+1	-1	-1	+1	+1	+1	---	1	0	3	0	0	0	3	0	4
												Sum	**74**	**0**	**45**	**0**	**0**	**1**	**44**	**1**	**36**

3. FORMULATION OF TRANSITION FREQUENCIES AND MATRICES

Using above transitions and transition frequencies given in Tables 2 to 11 the following transition frequency matrices are formed Medhi (1994).

Zone-I **Zone-II** **Zone-III** **Zone-IV** **Zone-V**

$$\begin{array}{c} \\ -1 \\ 0 \\ +1 \end{array}\begin{array}{c} \begin{array}{ccc} -1 & 0 & +1 \end{array} \\ \begin{bmatrix} 85 & 0 & 39 \\ 0 & 0 & 2 \\ 38 & 2 & 35 \end{bmatrix} \end{array} \quad \begin{array}{c} \\ -1 \\ 0 \\ +1 \end{array}\begin{array}{c} \begin{array}{ccc} -1 & 0 & +1 \end{array} \\ \begin{bmatrix} 95 & 1 & 38 \\ 1 & 0 & 1 \\ 38 & 1 & 26 \end{bmatrix} \end{array} \quad \begin{array}{c} \\ -1 \\ 0 \\ +1 \end{array}\begin{array}{c} \begin{array}{ccc} -1 & 0 & +1 \end{array} \\ \begin{bmatrix} 89 & 2 & 38 \\ 3 & 0 & 2 \\ 36 & 3 & 28 \end{bmatrix} \end{array} \quad \begin{array}{c} \\ -1 \\ 0 \\ +1 \end{array}\begin{array}{c} \begin{array}{ccc} -1 & 0 & +1 \end{array} \\ \begin{bmatrix} 103 & 0 & 32 \\ 0 & 0 & 1 \\ 31 & 1 & 33 \end{bmatrix} \end{array} \quad \begin{array}{c} \\ -1 \\ 0 \\ +1 \end{array}\begin{array}{c} \begin{array}{ccc} -1 & 0 & +1 \end{array} \\ \begin{bmatrix} 74 & 0 & 45 \\ 0 & 0 & 1 \\ 44 & 1 & 36 \end{bmatrix} \end{array}$$

Using the above transition frequencies, transition probability matrices are formed for each zone separately as follows Medhi (1994).

Zone-I

$$A_1 = \begin{bmatrix} 0.69 & 0 & 0.31 \\ 0 & 0 & 1 \\ 0.51 & 0.03 & 0.47 \end{bmatrix}$$

Zone-II

$$A_2 = \begin{bmatrix} 0.71 & 0.01 & 0.28 \\ 0.50 & 0 & 0.50 \\ 0.58 & 0.02 & 0.40 \end{bmatrix}$$

Zone-III

$$A_3 = \begin{bmatrix} 0.69 & 0.02 & 0.29 \\ 0.60 & 0 & 0.40 \\ 0.54 & 0.04 & 0.42 \end{bmatrix}$$

Zone-IV

$$A_4 = \begin{bmatrix} 0.76 & 0 & 0.24 \\ 0 & 0 & 1 \\ 0.48 & 0.02 & 0.51 \end{bmatrix}$$

Zone-V

$$A_5 = \begin{bmatrix} 0.62 & 0 & 0.38 \\ 0 & 0 & 1 \\ 0.54 & 0.01 & 0.44 \end{bmatrix}$$

Critically comparing Transition Probabilities it is observed that the probability for transitions from -1 to -1 is high in all the five zones. This implies that the depth of the Ground water level is shrinking year by year in all the five zones, this implies that year by year depth of the Ground water level is increasing and leading to a desert situation in this district. This probability is highest 0.76 in Kalyandurg Revenue Division and next to it is Penukonda Revenue Division is 0.71.

It is interesting to note that transitions from sate 0 to 0 are nil. It implies that retaining the same Ground water level year by year is not possible. It implies that continuous decay in the depth of the Ground water levels is present in this district. It implies that in all the years no hope of increasing the Ground water levelin this district. Rainfall is decreasing year after year and reaching to alarming famine conditions in this district.

Now we proceed to calculate limiting probabilities and steady state solutions if possible for the above five zones in the following section.

4. LIMITING PROBABILITIES AND CALCULATIONS OF STEADY SATE SOLUTIONS

Zone-I (Anantapuramu RD):

$$A_1 = \begin{bmatrix} 0.69 & 0 & 0.31 \\ 0 & 0 & 1 \\ 0.51 & 0.03 & 0.47 \end{bmatrix} A_1^2 = \begin{bmatrix} 0.63 & 0.01 & 0.36 \\ 0.51 & 0.03 & 0.47 \\ 0.59 & 0.01 & 0.41 \end{bmatrix} A_1^4 = \begin{bmatrix} 0.61 & 0.01 & 0.38 \\ 0.61 & 0.01 & 0.39 \\ 0.62 & 0.01 & 0.39 \end{bmatrix}$$

$$A_1^8 = \begin{bmatrix} 0.61 & 0.01 & 0.38 \\ 0.62 & 0.01 & 0.39 \\ 0.63 & 0.01 & 0.39 \end{bmatrix} A_1^{16} = \begin{bmatrix} 0.62 & 0.01 & 0.38 \\ 0.63 & 0.01 & 0.39 \\ 0.64 & 0.01 & 0.40 \end{bmatrix} A_1^{32} = \begin{bmatrix} 0.63 & 0.01 & 0.39 \\ 0.65 & 0.01 & 0.40 \\ 0.66 & 0.01 & 0.41 \end{bmatrix}$$

$$A_1^{64} = \begin{bmatrix} 0.66 & 0.01 & 0.41 \\ 0.68 & 0.01 & 0.42 \\ 0.69 & 0.01 & 0.43 \end{bmatrix} A_1^{128} = \begin{bmatrix} 0.73 & 0.01 & 0.45 \\ 0.75 & 0.01 & 0.46 \\ 0.76 & 0.01 & 0.47 \end{bmatrix} A_1^{256} = \begin{bmatrix} 0.88 & 0.01 & 0.54 \\ 0.90 & 0.01 & 0.56 \\ 0.92 & 0.01 & 0.57 \end{bmatrix} \text{...(4.1)}$$

Zone-II (Penukonda RD):

$$A_2 = \begin{bmatrix} 0.71 & 0.01 & 0.28 \\ 0.50 & 0 & 0.50 \\ 0.58 & 0.02 & 0.40 \end{bmatrix} A_2^2 = \begin{bmatrix} 0.67 & 0.01 & 0.32 \\ 0.65 & 0.02 & 0.34 \\ 0.65 & 0.01 & 0.33 \end{bmatrix} A_2^4 = \begin{bmatrix} 0.66 & 0.01 & 0.32 \\ 0.67 & 0.01 & 0.33 \\ 0.66 & 0.01 & 0.32 \end{bmatrix}$$

$$A_2^8 = \begin{bmatrix} 0.65 & 0.01 & 0.32 \\ 0.67 & 0.01 & 0.32 \\ 0.65 & 0.01 & 0.32 \end{bmatrix} A_2^{16} = \begin{bmatrix} 0.64 & 0.01 & 0.31 \\ 0.65 & 0.01 & 0.32 \\ 0.64 & 0.01 & 0.31 \end{bmatrix} A_2^{32} = \begin{bmatrix} 0.61 & 0.01 & 0.30 \\ 0.63 & 0.01 & 0.30 \\ 0.61 & 0.01 & 0.30 \end{bmatrix}$$

$$A_2^{64} = \begin{bmatrix} 0.56 & 0.01 & 0.28 \\ 0.57 & 0.01 & 0.28 \\ 0.56 & 0.01 & 0.28 \end{bmatrix} A_2^{128} = \begin{bmatrix} 0.48 & 0.01 & 0.24 \\ 0.48 & 0.01 & 0.24 \\ 0.48 & 0.01 & 0.24 \end{bmatrix} \dots (4.2)$$

Zone-III (Kadiri RD):

$$A_3 = \begin{bmatrix} 0.69 & 0.02 & 0.29 \\ 0.60 & 0 & 0.40 \\ 0.54 & 0.04 & 0.42 \end{bmatrix} A_3^2 = \begin{bmatrix} 0.64 & 0.03 & 0.33 \\ 0.63 & 0.03 & 0.34 \\ 0.62 & 0.03 & 0.35 \end{bmatrix} A_3^4 = \begin{bmatrix} 0.63 & 0.03 & 0.34 \\ 0.63 & 0.03 & 0.34 \\ 0.63 & 0.03 & 0.34 \end{bmatrix} \text{... (4.3)}$$

Zone-IV (Kalyandurg RD):

$$A_4 = \begin{bmatrix} 0.76 & 0 & 0.24 \\ 0 & 0 & 1 \\ 0.48 & 0.02 & 0.51 \end{bmatrix} A_4^2 = \begin{bmatrix} 0.69 & 0.00 & 0.30 \\ 0.48 & 0.02 & 0.51 \\ 0.61 & 0.01 & 0.40 \end{bmatrix} A_4^4 = \begin{bmatrix} 0.66 & 0.00 & 0.33 \\ 0.65 & 0.01 & 0.36 \\ 0.67 & 0.00 & 0.35 \end{bmatrix}$$

$$A_4^8 = \begin{bmatrix} 0.66 & 0.00 & 0.33 \\ 0.68 & 0.00 & 0.34 \\ 0.68 & 0.00 & 0.34 \end{bmatrix} A_4^9 = \begin{bmatrix} 0.66 & 0.00 & 0.33 \\ 0.68 & 0.00 & 0.34 \\ 0.68 & 0.00 & 0.34 \end{bmatrix} \text{... (4.4)}$$

Zone-V (Dharmavaram RD):

$$A_5 = \begin{bmatrix} 0.62 & 0 & 0.38 \\ 0 & 0 & 1 \\ 0.54 & 0.01 & 0.44 \end{bmatrix} A_5^2 = \begin{bmatrix} 0.59 & 0.00 & 0.40 \\ 0.54 & 0.01 & 0.44 \\ 0.57 & 0.00 & 0.41 \end{bmatrix} A_5^4 = \begin{bmatrix} 0.58 & 0 & 0.40 \\ 0.57 & 0 & 0.40 \\ 0.57 & 0 & 0.40 \end{bmatrix}$$

$$A_5^8 = \begin{bmatrix} 0.56 & 0 & 0.39 \\ 0.56 & 0 & 0.39 \\ 0.56 & 0 & 0.39 \end{bmatrix} \ldots (4.5)$$

5. CONCLUSIONS

It is observed that from the above steady state solutions for zone-I and zone-IV have no steady state solutions are observed and hence the states of the Markov chain namely -1, 0, +1 are recurrent states which implies that Ground water level in this zones are water facilities are available other than Rain water like HLC, Srujala Sravanthi water lifting irrigation project and so on this leading to recurrent states in these two zones.

Whereas in other three zones steady state solutions are obtained and limiting probabilities are given in equations 4.2, 4.3 and 4.5 respectively. Critically comparing limiting probabilities the state -1 i.e. increasing the depth of the Ground water level is high in Kadiri Revenue Division and next to it is Dharmavaram Revenue Division and least in Penukonda Revenue Division. This implies that famine conditions will appear more faster in Kadiri Revenue Division than other two Revenue Divisions namely Penukonda and Dharmavaram. Since this probabilities are approximately equal to 0.5 or more there is an immediate and necessity to take action in these zones to improve water facilities so that one can avoid famine conditions in these zones. Rescue operations are immediately suggested in these zonesof Anantapuramu and Kalyandurg Revenue Divisions because the probability of arriving Famine conditions in these zones are very high when compared to other zones.

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