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A INTERESTING RESULT IN PROBABILITY

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The following problem is of interest and attracted our attention.

Problem: Player 'A' and player 'B' have respectively n and $(n + 1)$ unbiased coins. They toss their coins. Suppose 'x' and 'y' denote respectively the number of heads obtained by the player A and B. If $x \geq y$ player A wins, whereas if $x < y$, B is the winner.

Then we can show that for every n, P [A wins] = P[B wins] = 0.5.

We give the generalization of the above problem, which is as follows.

Generalized Problem: Player 'A' and 'B' have respectively n and $(n + m)$ unbiased coins. When these players complete tossing of their coins, let x and y denote respectively the number of heads obtained by the player A and B.

Player A wins if $x + p \geq y$ and player B wins if, $x + p < y$. Find m and p such that, for every n,

$$P\,[A \text{ wins}] = P\,[B \text{ wins}] = 0.5.$$

The following theorem gives the solution of the above-generalized problem.

Theorem: Let player A and B have n and $(n + 2r + 1)$ unbiased coins. They toss their coins. Let x and y denote the number of heads obtained by player A and B respectively.

A wins if $x + r \geq y$ and player B wins if $x + r < y$. Then for every n and r,

$$P\,[A \text{ wins}] = P\,[B \text{ wins}] = 0.5.$$

Proof: We first evaluate the probability of player A winning the game as follows.

$$\text{P [A wins the game]} = \sum_{x=0}^{n} {}^{n}C_x \left(\frac{1}{2}\right)^n \sum_{y=0}^{x+r} {}^{n+2r+1}C_y \left(\frac{1}{2}\right)^{n+2r+1}$$

$$= \frac{1}{2^{2n+2r+1}} \sum_{x=0}^{n} {}^{n}C_x \sum_{y=0}^{x+r} {}^{n+2r+1}C_y$$

$$= \frac{1}{2^{2n+2r+1}} \sum_{x=0}^{n} {}^{n}C_{n-x} \left[{}^{n+2r+1}C_{n+2r+1} + {}^{n+2r+1}C_{n+2r} + \ldots + {}^{n+2r+1}C_{n-x+r+1}\right]$$

$$= \frac{1}{2^{2n+2r+1}} \sum_{x=0}^{n} {}^{n}C_{n-x} \sum_{y=n-x+r+1}^{n+2r+1} {}^{n+2r+1}C_y$$

$$= \sum_{x=0}^{n} \text{P [}A\text{ obtains } (n\text{-}x) \text{ heads and } B \text{ obtains more than } (n\text{-}x\text{+}r) \text{ heads]}$$

$$= \text{P [B wins the game]}$$

Thus P [Player A wins] = P [Player B wins] $= \dfrac{1}{2}$

Remarks: It can be observed that for $r = 0$, the generalized problem reduces to the original basic problem stated earlier. Hence the result.

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